

# $Z(2)$ vortices and the string tension in $SU(2)$ gauge theory\*

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We exhibit the appropriate variables allowing the plus-minus ( $Z(2)$ ) ‘reduction’ of the Wilson loop operator which provides a direct measure of the thin, thick and ‘mixed’  $Z(2)$  topological, gauge-invariant vortices in  $SU(2)$  LGT. Simulations with the Wilson action, as well as a perfect action smoothing procedure, show the string tension to be reproduced from the contributions of these excitations.

We report on recent work on the crucial role of vortices characterized by  $Z(N)$  flux for maintaining confinement at weak coupling in  $SU(N)$  gauge theory. The present work is a continuation of our on-going project over the last several years [1].

We begin by distinguishing between various types of vortices. Recall that in the continuum formulation there is no local distinction between pure  $SU(N)$  and  $SU(N)/Z(N)$  gauge theories, but in the lattice formulation there is. Now for continuum  $SU(N)/Z(N)$  fields, vortices are topologically classified by  $\pi_1(SU(N)/Z(N)) = Z(N)$ . A vortex forms a closed 2-dim structures in  $d = 4$ . Topologically, it is also possible to have Dirac monopoles, also classified by  $\pi_1(SU(N)/Z(N))$ . The Dirac sheet of such a monopole loop ( $d = 4$ ) may be described as defining a ‘punctured’ vortex. On the lattice the  $SU(N)$  and  $SU(N)/Z(N)$  theories differ by  $Z(N)$  degrees of freedom. Exciting these  $Z(N)$  degrees of freedom gives rise to ‘thin’  $Z(N)$  vortices. They are very efficient at disordering at small  $\beta$ ; but are directly suppressed by the  $SU(N)$  plaquette action and become unimportant at large  $\beta$ . This reflects the fact that the distinction between  $SU(N)$  and  $SU(N)/Z(N)$  LGT must disappear as the con-

tinuum is approached.

Failure to properly distinguish between thick and thin, i.e. between the (lattice analogs of the)  $\pi_1(SU(N)/Z(N)) = Z(N)$  excitations arising from the  $SU(N)/Z(N)$  part of the  $SU(N)$  gauge group versus the  $Z(N)$  excitations of the  $Z(N)$  part of the group, has caused confusion in the lattice literature. A clean separation can be achieved by introducing new separate  $SU(N)/Z(N)$  and  $Z(N)$  variables. We treat explicitly the  $N = 2$  case which is the actual case of our numerical simulations; the extension to general  $N$  is straightforward. The original  $SU(2)$  LGT is given in terms of the bond variables  $U_b \in SU(2)$  residing on bonds  $b$ . The new variables are coset bond variables  $\hat{U}_b \in SU(2)/Z(2) \sim SO(3)$ , and  $Z(2)$  variables  $\sigma_p \in \{\pm 1\}$  residing on plaquettes  $p$ . The  $SU(2)$  theory, i.e. the partition function and all correlations, can be rewritten in terms of the variables  $\hat{U}_b$  and  $\sigma_p$  [1]. This rewriting is exact and gauge invariant and expresses the partition function as a coupled  $SO(3) - Z(2)$  system over the Haar invariant measures of the two groups. The  $SU(2)$  plaquette action in the new variables becomes the product of a non-negative function of the  $\hat{U}_b$ ’s times  $\sigma_p$ . Thus the sign of the  $SU(2)$  action on a plaquette  $p$  is now simply given by the variable  $\sigma_p$ . The integration measure in the new variables contains a constraint that requires that the product of the  $\sigma_p$ ’s over the faces of a cube equals  $-1$  if the cube is the site of a Dirac

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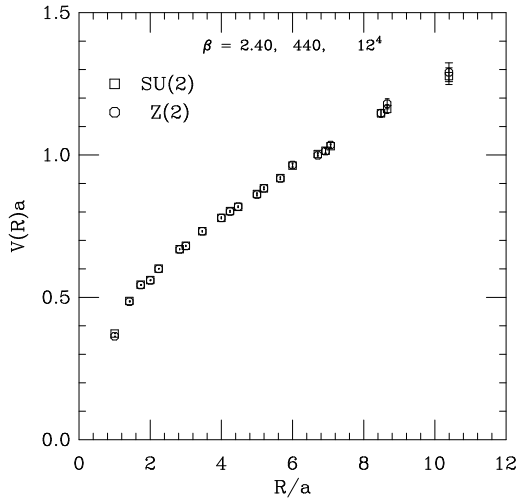


Figure 1. The heavy quark potential measured on an ensemble of 440  $12^4$  configurations with Wilson action at  $\beta = 2.4$ .

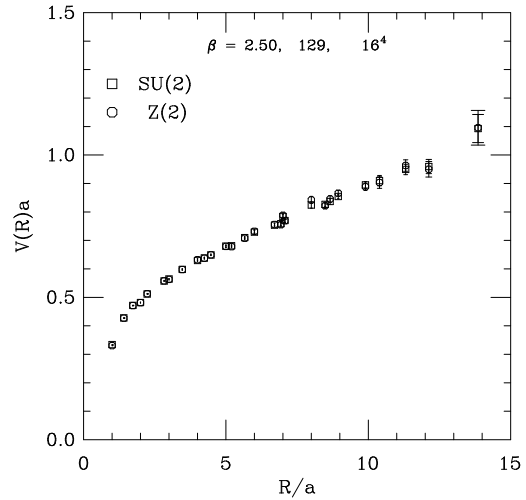


Figure 2. The heavy quark potential measured on an ensemble of 129  $16^4$  configurations with Wilson action at  $\beta = 2.5$ .

monopole in the  $\{\hat{U}\}$  configuration.

The new variables allow an immediate identification of the various possible excitations. Thin vortices are created by the excitation of the  $\sigma_p$  variables. They are necessarily localized to one lattice spacing thickness, and incur a direct action cost proportional to their vortex sheet area. At large  $\beta$  then, long thin vortices are suppressed, and only short thin vortices survive. ‘Thick’ vortices are vortices in the  $\{\hat{U}_b\}$  configurations. There is no negative plaquette action suppression associated with them, so that, by being sufficiently spread out, these vortex configurations can cost locally very little action even at large  $\beta$ ; while, by the very gradual variation of the bond variables, can disorder the system over long scales. ‘Punctured’ thick vortices can also exist at large  $\beta$  if the ‘hole’, whose boundary is a Dirac monopole current loop, is small (of the order of one lattice spacing for sufficiently large  $\beta$ ). This is because the hole has to be covered by an open thin vortex sheet to satisfy the above constraint on the cubes occupied by the monopoles forming its boundary. The result is a ‘hybrid’ vortex. Hybrid vortices, whose presence was explored in [1],

act then in the same way as pure thick vortices to disorder over long scales.

Various observables also acquire a physically transparent form when expressed in terms of the new variables. The Wilson loop, in particular, is revealed as essentially a vortex counter. It becomes the product of an  $SO(3)$  part and a  $Z(2)$  part that manifestly exhibit the flip in sign whenever any of the three types of vortices links with the loop. What is surprising, however, is that at weak coupling, apart from possible edge (i.e. perimeter) contributions, this sign fluctuation appears to be all that there is. This suggests that the linear piece, i.e. the string tension, if nonvanishing, comes *entirely* from the expectation of the sign flip counting vortices, a rather remarkable assertion. We checked this by numerical measurement. We compared the heavy quark potential extracted from the full Wilson loop operator expectation to that extracted from the expectation of its sign. First using the Wilson action, results of computation of these two quantities, labelled  $SU(2)$  and  $Z(2)$ , respectively, are shown in figures 1, 2. There is no discernible difference between the two curves. Note that this coincidence

extends down to small Wilson loops. This is because we are including *all* vortices, i.e. also thin ones that can contribute to the area law piece of small loops. We next checked this by performing the computation using a perfect action with a smoothing procedure based on the RG [2]. The point is that smoothing removes short distance fluctuations while preserving long distance physics - in particular, the string tension of the full Wilson loop remains unchanged under smoothing. A necessary test then of any claim concerning long distance physics (here, the claim that replacing the full Wilson loop operator by its sign gives the same string tension) is that it remain invariant under smoothing. This is a highly non-trivial test since, in general, the smoothed configurations are very different from the original unsmoothed ones. The results are shown in figures 3 and 4 for one and three smoothing steps, respectively. Note that with increasing smoothing there is, as it should be, increasing deviation at short distances, and in the right direction (all thin and generally short vortices are eliminated). But the coincidence of the long distance  $Z(2)$  and full potentials, i.e. the string tension for large loops, remains invariant. This is the regime of long thick and hybrid vortices at large  $\beta$ . It should be noted that there is a very delicate cancellation between positive (even number of vortices) and negative (odd number) contributions in the  $Z(2)$  expectation that conspires to reproduce the asymptotic string tension. Conversely, it can be checked that eliminating all (odd numbers of) vortices linked with the loop eliminates the linear potential.

Closely related results are reported in [3]. We thank J. Greensite for discussions.

## REFERENCES

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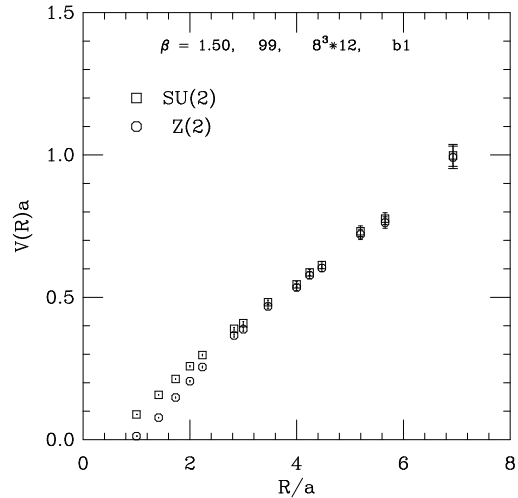


Figure 3. The heavy quark potential measured after 1 smoothing step on an ensemble of  $99\ 8^3 \times 12$  configurations generated with a fixed point action at  $\beta = 1.5$  which corresponds to very nearly the same physical lattice spacing as  $\beta = 2.4$  for the Wilson action.

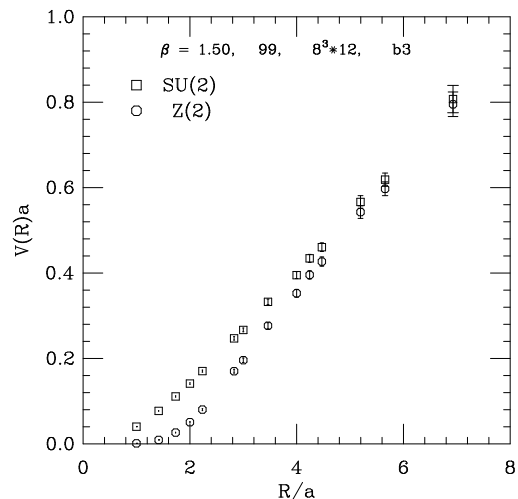


Figure 4. The heavy quark potential measured after 3 smoothing steps on an ensemble of  $99\ 8^3 \times 12$  configurations generated with a fixed point action at  $\beta = 1.5$ .